Analysis of Andres Galarraga's Home Run of May 31, 1997

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ABSTRACT

In this article the home run hit by Andrés Galarraga at the Florida Marlins' home stadium in 1997 is analyzed. Assigned initially at 529, feet it was considered one of the longest in major league baseball history, but the distance estimate was later lowered to 468 feet. A mathematical model is developed to determine the trajectory of the ball using known principles of physics. The reliability of the model is demonstrated by comparisons with actual trajectory data measured by Statcast. The authors combine physics, descriptive geometry, detailed video analysis, and remote sensing data to examine Galarraga's home run. A breakthrough emerged from utilizing a high-resolution (LIDAR technology) map of Pro Player Stadium allowing the determination of accurate horizontal and vertical coordinates of the ball's impact point on the seats of the stadium. To account for uncertainties, eighteen cases were considered by varying the initial conditions based on historical ranges of MLB home runs, wind speed, and direction. Using orthogonal and conical projections, the most reliable solutions were selected by comparing the maximum height of the ball for each case to the actual height reached by the ball as shown on the video frame. The results show that Galarraga's home run reached a distance between 517.5 and 529.4 feet, with 524 feet the most probable value. Therefore it is one of the few home runs prior to the Statcast era to be proven to have exceeded the 500-foot distance.

1. INTRODUCTION

On May 31, 1997, Andres Galarraga ("The Big Cat") of the Colorado Rockies stepped up to the plate at Pro Player Stadium, in Miami, Florida, to confront Florida Marlins pitcher Kevin Brown in the fourth inning with the bases loaded. Galarraga connected on a 2–2 hanging slider to hit a mammoth grand slam to left-center field, into the twentieth row of the upper deck. Its distance was initially estimated at 573 feet and then changed to 529 feet (See Figure 1, from the *Rocky Mountain News*[1]). Galarraga's home run was considered one of the longest homers in the history of major league baseball and one of the few to have exceeded 500 feet in distance.[2]

However, Greg Rybarczyk—founder of Home Run Tracker—studied some of the longest home runs in MLB and found that many of their distances had been overestimated.[3] In particular, he reduced the distance of Galarraga's homer to 468 feet, excluding Galarraga from the elite 500-foot-plus group. The objective of this research is to resolve this conflict and answer these questions: What was the true distance of Galarraga's homer? Is this homer one of the few that has exceeded the distance of 500 feet?

2. MATHEMATICAL MODEL TO ANALYZE THE TRAJECTORY OF THE BATTED BALL

2.1 Dynamics equations

Newton's second law states that the sum of the gravitational ($\vec{F}_g$), drag ($\vec{F}_d$) and Magnus ($\vec{F}_m$) force vectors

ROCKY MOUNTAIN NEWS

Figure 1. *The Rocky Mountain News* reports a 529-foot distance a day after Galarraga's homer on June 1, 1997, in Denver, Colorado.

is equal to $m\vec{a}$, for a batted ball of mass m that has acceleration $\vec{a}$:

$$\vec{F}_g + \vec{F}_d + \vec{F}_m = m\,\vec{a} \qquad (1)$$

where the Coriolis force due to earth's rotation has been neglected. The gravity force is given by $\vec{F}_g = -\,m.g\hat{k}$ where $\hat{k}$ is a unit vector along the global fixed axis z in the inertial reference frame (x, y, z). The drag force is given by:

$$\vec{F}_d = -\frac{1}{2}\,\rho\, C_d A\, V \vec{V} \qquad (2a)$$

$$C_d = 0.50 - \frac{0.227}{1 + e^{-\left(\frac{1.467\,V - 108.7}{21.0}\right)}} \qquad (2b)$$

where ρ is the air density, C_d is the drag coefficient, A is the cross-sectional area of the ball, $\vec{V}$ is the relative velocity vector given by the difference between the ball's and wind's velocity vectors with respect to ground, V is the speed or modulus of $\vec{V}$ and C_d is the drag coefficient.[4] In Equation (2b) the speed V is given in miles per hour (mph).

2.2 The Magnus force and the rotational velocity components

The Magnus force is given by:

$$\vec{F}_m = \frac{1}{2}\,\rho\, C_m A\, V^2\,(\hat{\omega} \times \hat{V}) \qquad (3)$$

where $\hat{\omega}$ and $\hat{V}$ are the unit vectors of the rotational velocity vector $\vec{\omega}$ and the translational velocity vector $\vec{V}$, respectively, and C_m is the Magnus coefficient that depends on the spin factor S. The following expression of C_m has been proposed by Sawicki et al. in 2013 and cited in Nathan for its good fit to experimental measurements:[5]

$$C_m = 1.5\,S \quad \text{for } S \le 0.1 \qquad (4a)$$

$$C_m = 0.09 + 0.6\,S \quad \text{for } S > 0.1 \qquad (4b)$$

The spin factor is given by $S = 0.00853\,\frac{\omega}{V}$ where ω is the rotational speed in rpm and V is in mph. The decay of the rotational speed ω can be incorporated by multiplying the spin factor S by κ^t where κ is the spin decay factor and t is the time in seconds. For example, a value of $\kappa = 0.98$ means that the rotational speed decays 2% each second.

The modulus F_m of the Magnus force in Eq. (3) is given by Eq. (5) where γ is the angle between the $\vec{\omega}$ and $\vec{V}$ vectors:

$$F_m = \frac{1}{2}\,\rho\, C_m A\, V^2 \operatorname{sen}\gamma \qquad (5)$$

$$\gamma = \arccos(\vec{\omega}\cdot\vec{V})/\omega V \qquad (6)$$

where the dot · means the scalar product of vectors $\vec{V}$ and $\vec{\omega}$. In the fixed global coordinate system (x, y, z), these vectors are given in terms of its scalar components and the unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ along the coordinates x, y, z, respectively:

$$\vec{V} = V_x\,\hat{i} + V_y\,\hat{j} + V_z\,\hat{k} \qquad (7)$$

$$\vec{\omega} = \omega_x\,\hat{i} + \omega_y\,\hat{j} + \omega_z\,\hat{k} \qquad (8)$$

In the local coordinate system, the rotational velocity vector $\vec{\omega}$ can be written in terms of the time-varying orthonormal vectors $\hat{e}_b$, $\hat{e}_s$, $\hat{e}_g$ and the spin rates ω_b, ω_s, ω_g which are defined as the back, side and gyro spin components of the rotational speed, respectively:

$$\vec{\omega} = \omega_b\,\hat{e}_b + \omega_s\,\hat{e}_s + \omega_g\,\hat{e}_g \qquad (9)$$

$$\hat{e}_b = \hat{V} \times \hat{k} \qquad (10a)$$

$$\hat{e}_s = \hat{e}_b \times \hat{V} \qquad (10b)$$

$$\hat{e}_g = \hat{e}_s \times \hat{e}_b = \hat{V} \qquad (10c)$$

Note that ω_b could be positive or negative if it produces upward movement (backspin) or downward movement (topspin), respectively.

In matrix form, the global rotational speed components (ω_G) are related to the local ones (ω_L) by the transformation matrix **A** which is an orthogonal matrix. They are written in compact form (Eq. 11) and expanded form (Eq. 12) as follows:

$$\boldsymbol{\omega_G} = \mathbf{A}\,\boldsymbol{\omega_L} \qquad (11)$$

$$\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix} V_y/V_{xy} & -V_xV_z/VV_{xy} & V_x/V \\ -V_x/V_{xy} & -V_yV_z/VV_{xy} & V_y/V \\ 0 & V_{xy}/V & V_z/V \end{bmatrix} \cdot \begin{bmatrix}\omega_b\\ \omega_s\\ \omega_g\end{bmatrix} \qquad (12)$$

where $V_{xy} = (V_x^2 + V_y^2)^{1/2}$. The inverse relation is given by Eq. (13) where $\mathbf{A^t}$ is the transpose of **A**:

$$\boldsymbol{\omega_L} = \mathbf{A^t}\,\boldsymbol{\omega_G} \qquad (13)$$

Given the initial values of the rotational components ω_b, ω_s, ω_g of the batted ball, the initial rotational components ω_x, ω_y, ω_z in the global coordinates are calculated from Eq. (12). Neglecting the spin decay, angular momentum is conserved during the ball's flight, and therefore the rotational velocity vector $\vec{\omega}$ remains unchanged in the inertial (x, y, z) reference frame. However, due to the change in the direction of the velocity vector $\vec{V}$ the local rotational frame is not an inertial frame and the rotational components ω_b, ω_s, ω_g may change; they are calculated from Eq. (13) at each time step. If the spin decay (κ) is incorporated in the ball's flight, each rotational component ω_b, ω_s, ω_g and ω_x, ω_y, ω_z is multiplied by κ^t. The set of three non-linear differential equations (Eq. 1) are solved by finite differences considering small time increments.

2.3 Validation of the Model

The model has to be validated before applying it to analyze the home run of Galarraga. Validation is done in two steps. First, neglecting air resistance and Magnus effect, the results of the model are compared to the analytical solution obtained from the classic theory of projectile motion. For a batted ball, given an initial height h_0 = 3.28 ft, an exit speed V_0 = 110 mph and a launch angle θ = 35°, the analytical solution gives the following results for the maximum height (H) and range (D) of the ball: H = 136.31 ft at t = 2.875 s and D = 764.6 ft at t = 5.786 s. Results for the simulation model using a time step of 0.001 s leads to values of H = 136.35 ft at t = 2.875 and D = 764.7 ft at 5.786 s. The very small differences in height and range are attributable to numerical discretization errors.

Next the model is validated by comparison with actual trajectory data measured by the Statcast system on a fly ball hit by Kris Bryant in a 2016 NLCS game, reported by Alan Nathan.[6] The game took place at Dodger Stadium on October 20, 2016. According to Nathan, Statcast data show that the ball left the bat with an exit speed of V_0 = 107.1 mph, a vertical launch angle of θ = 20.8° and an initial spray angle of β_i = 2.4° to the left of dead center field. The ball landed 382.6 feet from home plate at a final spray angle of β_f = 4.4° to the right of dead center field with a hang time of t = 3.9 s. The complete trajectory of the ball in vertical and horizontal planes is depicted graphically in Nathan's paper; the horizontal plane (top view) of the ball shows a curvilinear trajectory, indicating that the batted ball had a strong component of side spin (there is no wind) in addition to the back spin typical of long fly balls. These Statcast data provide us a unique opportunity to test the validity of the simulation model subjected to three-dimensional motion. The elevation of Dodger Stadium is reported to be 515 feet above sea level. According to the box score the game started at 5:09 PM and Bryant's fly ball occurred at the top of the fifth inning. We estimated the time to be approximately 6:40 PM. At that moment Weather Underground at the nearest station indicated an air temperature of 80.96° F, a barometric pressure of 1010.9 hPa and a dew point of 28.04° F with no wind. Using these data the calculated air density was 0.073 lb/ft^3.

To run the model, additional information is required regarding the exit spin rate of the ball, but this is not presented in Nathan's paper (spin data of batted balls are not published by Statcast). Observation of the video shows that Bryant's bat is tilted down at approximately 45 degrees at the moment of the impact so we assume that the ball leaves the bat with similar amount of backspin and sidespin. Thus the problem is formulated in the following way: given the known initial conditions V_0, θ and β_i the goal of the simulation is to adjust the values of the initial spin rates ω_b and ω_s in order to replicate as accurately as possible the ball's trajectory measured by Statcast. To be able to compare the results of the model to the measured data we adopt the same system of coordinates as the one shown in Nathan's paper. So the coordinate system has its origin in the back of home plate, where the x-axis points to the catcher's right, the y-axis points to the second base and the z-axis points in the vertical direction. According to this system the initial position of the batted ball is estimated from Nathan's charts as x_o = 0, y_o = 2 ft and z_o = 3 ft. The numerical solution was calculated with a time interval of 0.001 seconds. After some iterations with different spin rates, the best results were obtained with ω_b = 830 rpm and ω_s = 820 rpm. A small spin decay ($\kappa = 0.98$) was used.

The results of the model are compared to the measured data in Figure 2. The scales have been distorted in the figures to allow small differences to be appreciated. The Distance in Figure 2 (right) is the horizontal distance from the origin of coordinates. The measured data from Statcast are obtained by digitizing the ball's trajectory from Nathan's figures. The side and top views in Figure 2 show a good correspondence. The model yields a horizontal distance of 382.6 ft at t = 4.0 s and a final spray angle of β_f = 4.4°, as compared to 382.6 ft at t = 3.9 s and β_f = 4.4° of Statcast. It is concluded that the model reproduces satisfactorily the measured data.

The numerical simulation allows examining in more detail the behavior of the spin rates during the flight. Time variations of ω and its components ω_b, ω_s, and ω_g are shown in Figure 3.

A gradual reduction in ω_b and ω_s is compensated by a gradual increase in ω_g during the first two seconds of the ball's flight, meaning that part of the back and side spins are being converted into gyro spin. A more pronounced decline in ω_s starts developing after the ball reaches its peak at t = 1.87 s. Surprisingly, the gyro spin component of the rotational speed is greater than the back and side spin components at the end of the flight. The velocity vector is no longer perpendicular to the rotational velocity vector and the angle between them (Ec. 6) evolves from an initial value of 90° to a value of 44.2° at the end of the trajectory at t = 3.96 s. Table 1 shows the initial and final values of the different components of the spin rates. The slight reduction in ω is due to the spin decay factor used in the simulation.

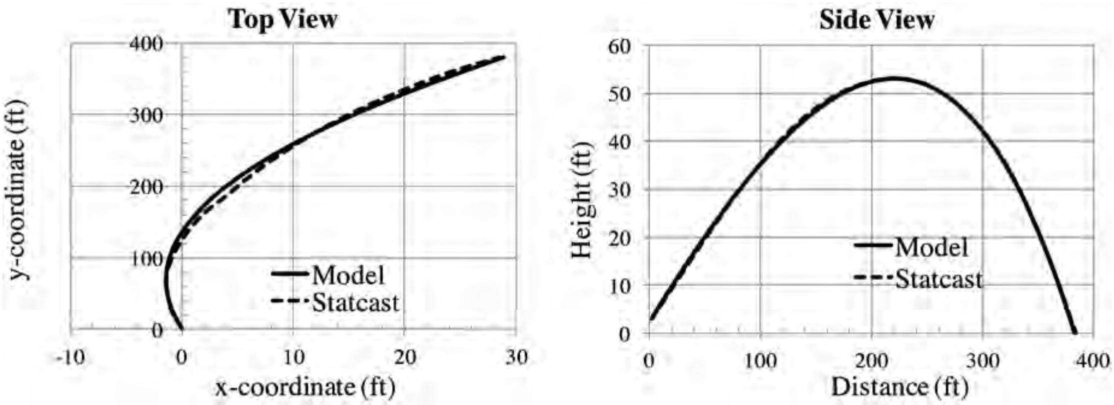


Figure 2. Model and Statcast (Nathan, 2017) trajectories of the Kris Bryant's fly ball, shown in horizontal (left) and vertical (right) planes.

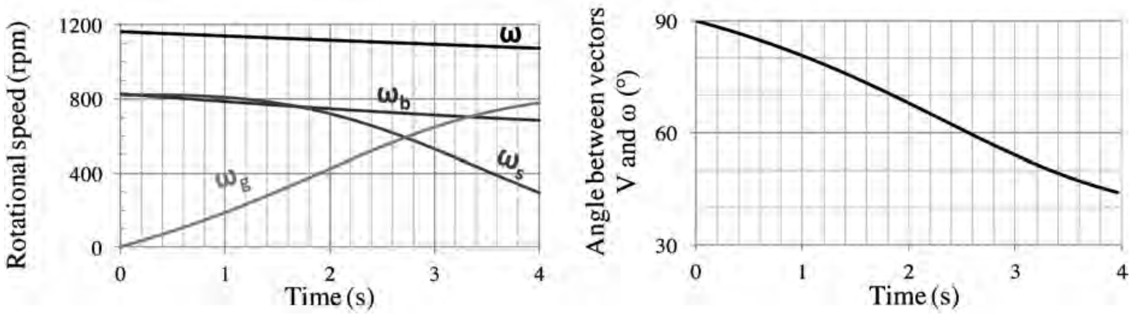


Figure 3. Time variation of the calculated values of the spin rates (left) and of the angle between vectors $\vec{V}$ and $\vec{\omega}$ (right), during the ball's flight.

Initial Values: Speed Rates (rpm)				Initial Values: γ (º)	Final Values: Speed Rates (rpm)				Final Values: γ (º)
ω	ω_b	ω_s	ω_g		ω	ω_b	ω_s	ω_g	
1167	830	820	0	90	1077	685	306	773	44

Table 1. Initial and final values of the spin rates during Kris Bryant´s fly ball. γ is the angle between vectors $\vec{V}$ and $\vec{\omega}$.

3. Analysis of Galarraga's Home Run

There are two categories of unknowns that must be first determined in order to calculate the total distance traveled by the home run. The first category refers to the flight time (t_v), the coordinates (d, h) of the point where the ball landed on the stadium and the weather conditions (air density and wind speed and direction). The second category of unknowns refers to the initial conditions at the moment that the bat hits the ball. These are the initial translational speed (V_0), the vertical launch angle (θ), the horizontal spray angle (β) and the rotational speed components (back spin ω_b and side spin ω_s, assuming an initial gyro spin $\omega_g = 0$); these variables were not measured in 1997 but must be estimated in some way in order to solve the problem. The unknowns in this second category have a higher level of uncertainty than those of the first category.

3.1 Flight time and impact point

A frame-by-frame analysis of FOX video (MLB, 2016) was performed with a professional video editing program (Adobe Premiere). The flight time is the time from the moment the ball is batted to the moment when the ball hits the stadium. The internal chronometer of the editing program indicates a time flight of 4.67 seconds, which differs from the 4.97 seconds given by ESPN's Home Run Tracker (2016).[3] Given this discrepancy, two independent chronometers were inserted into the program; both yielded 4.67 seconds, therefore this time was adopted as the flight time. The ball's point of impact was located in the middle of row 20 on the third sector of the upper deck (from the LF line) which has 30 rows (Figure 4, left).

The coordinates of the ball's impact point and of home plate were determined using LIDAR technology (Laser Imaging Detection and Ranging) which gives a high-resolution 3D map of the Pro Player Stadium. For that purpose, a point cloud image of the stadium was downloaded from the US Geological Survey website and processed with Global Mapper software.[7] The impact point was placed according to the location provided by the video frame (Figure 4, left). Home plate was located using the distances measured by physicist Brian Raue from the plate to the LF and RF fences, which were 327.5 feet and 347.25 feet, respectively, as shown in Figure 4, right.[8] A horizontal distance (d) of 413.1 feet from the back of home plate and a height (h) of 97.5 feet above field level were obtained (Figure 4, right). The line connecting the plate to the point of impact forms an angle of 14.4° to the LF line.

3.2 Weather conditions

The game's box score indicated a temperature of 87° F on May 31, 1997, a cloudy sky without precipitation, and a game start time of 1:17 PM with a duration of 3:32.[9] Climatic conditions included average relative humidity of 72% with a maximum of 97% and sea-level pressure of 1013 hPa.[10] The altitude is 1 m. An air density of 1.148 kg/m^3 was calculated. The box score indicated winds of 6 mph out to center field at the beginning of the game. However, an analysis of the box score suggests that the home run could have taken place sometime between 2:40 PM and 3:00 PM. At 2:40 PM the closest weather station (KOPF, about 4 miles from the stadium) indicates recorded winds of 11 mph at an angle of about 11° out to the left of the LF line and around 3:00 PM recorded winds of 13 mph from right to left which coincide with the values used by ESPN.[11] Given the uncertainties regarding the wind, these three values were used in the calculations.

3.3 Initial conditions

It can be seen in the video that the bat made contact with the ball at approximately 3 feet (vertical distance) above field level. We also assume that the ball was hit at 2 feet (horizontal distance) from the back of the plate, thus setting the initial position of the batted ball. Since the other initial conditions for the ball (V_0, θ, ω_b, and ω_s) are unknown, they must be assumed. Although there are many possible solutions to the problem, it is not necessary to adopt totally arbitrary values since in the last two years Statcast has measured the initial speed (V_0) and vertical launch angle (θ) of home runs hit in MLB. This information is used in this work to define ranges of possible values for these variables. An analysis of MLB's data indicates that the 50 longest home runs connected in 2015 and 2016 had an initial speed between 101 and 119 mph and a launch angle between 18° and 45°.[12] Therefore, these ranges of values for V_0 and θ are adopted for the calculations. The adopted range of horizontal exit spray angle (β) is 5° to 20° measured from the LF line (Figure 4, right); the actual value depends on the wind direction and the amount of side spin.

Rotational speeds (not reported by Statcast) have been obtained indirectly by Nathan.[13] For a set of 281 homers the values found were between 650 rpm and 3500 rpm. Homers that reached greater distances (D > 450 ft) had values above 1100 rpm. Therefore the range of ω values considered in the calculation was 1000 rpm to 3500 rpm. Since the ball was pulled to left field, it is assumed that there is a sidespin rotational component that causes the ball to break toward the LF foul pole based on Statcast data.[14] The rotational component of backspin, sidespin and gyrospin speeds

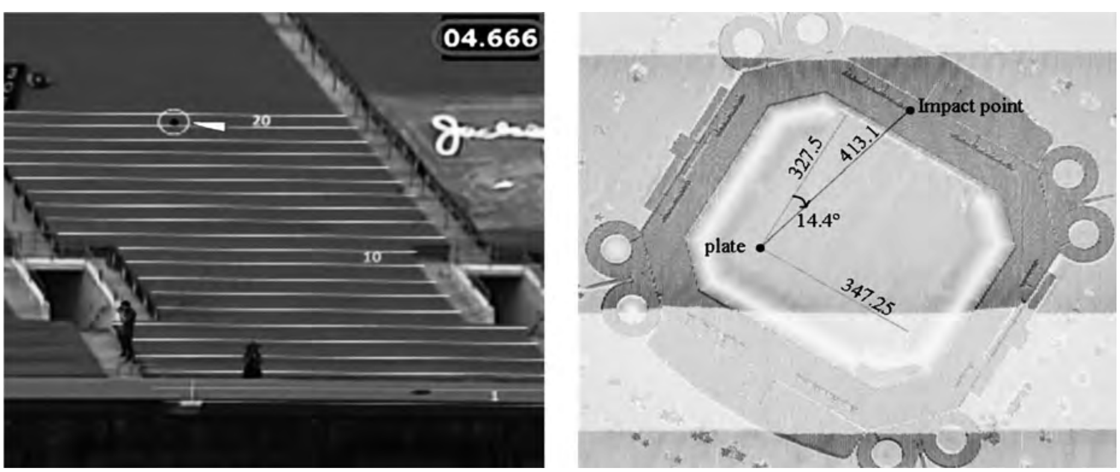


Figure 4. (Left) Video frame showing the ball´s impact point on row 20 at a time of 4.67 seconds. The frame has been modified to highlight the rows that were covered by a blanket. (Right) Image from LIDAR point clouds showing the distances from the home plate to the impact point and to the LF and RF fences.

are assumed to be 0.94 ω, 0.34 ω and 0.00 ω, respectively being ω the exit rotational speed, considering that at the moment of impact the bat was tilted downward at an angle of about 20° to the horizontal plane. The influence of other possible values for the rotational components will be discussed in the next section. A small value of spin decay was assumed ($\kappa = 0.98$), which means that the rotational speed decays 2% each second.

3.4. Solutions

The problem to solve is to find a trajectory such that at 4.67 seconds the ball is at the observed point of impact in the stadium. This point is defined by the height h = 97.5 ft above field level and the horizontal distance d = 413.1 ft from the home plate (Figure 4, right). For each set of initial conditions, the trajectory of the ball is obtained solving the set of non linear differential equations (Eq. 1) by finite differences using a time increment of 0.005 s. The problem is not expected to have a unique solution.

Given the uncertainties regarding the wind speed and direction, three wind hypotheses were considered as mentioned earlier:

1) 6 mph out to center field (as recorded in the box score)
2) 11 mph at 11° out to the left of the LF line (KOPF, 2:40PM), and
3) 13 mph from right to left (KOPF, 3:00PM).

For each wind hypothesis the rotational speed (ω) was varied in increments of 500 rpm between 1000 and 3500 rpm thus generating a total of 18 cases (Table 2). Other initial conditions were varied within the selected ranges to find trajectories for which the calculated point of impact of the ball in the stadium matched the observed point at 4.67 seconds.

Table 2 shows the results for the 18 cases analyzed; there are 6 cases for each wind hypothesis. The initial conditions (ω, V_0, θ, β), the maximum height (H) reached by the ball and its time of occurrence (t_{max}) and the total distance (D) measured from the point where the bat strikes the ball, are shown. The shaded rows in Table 2 indicate the seven most reliable solutions that are indentified in the next section. The relative error of the impact point predicted by the model is less than 0.25 per thousand for all cases. It should be pointed out in Table 2 that the total distance traveled (D) turned out to be above 500 ft for all cases.

It should be noted that the solution depends on the magnitude and the sign of the sidespin adopted. Another set of 18 cases were also generated changing the sign of the sidespin, keeping its magnitude, so that the ball breaks from left to right, although this hypothesis has a very small probability of occurrence according to data measured by Statcast (Figure 4 in Nathan 2017[6]). For this assumption the results indicate a small variation in the total distance (D) as compared to the results shown in Table 2, less than 2%, and again all cases exceed 500 feet.

The solutions shown in Table 2 were generated assuming rotational backspin (ω_b) and sidespin (ω_s) components of 0.94ω and 0.34ω, respectively. Another set of solutions were additionally generated assuming $\omega_b = 0.80\omega$ and $\omega_s = 0.60\omega$; the results indicate a small reduction of less than 0.5% in the total distance (D) for the 18 solutions shown in Table 2.

When the analysis is carried out in the absence of wind, the results indicate a reduction of less than 3% in the total distance (D) for all the cases shown in Table 2; furthermore, all of the seven most reliable solutions exceed 500 feet.

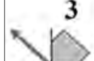
Wind Hypothesis	Solution #	Exit Rotational Speed ω (rpm)	Exit Speed V_0 (mph)	Exit Launch Angle θ (degrees)	Exit Spray Angle β (degrees)	Max. Height H (ft)	Time of Max. Height t_{max} (sec)	Total Distance D (ft)
1  6 mph	1	1000	118.8	35.6	16.4	138.7	2.9	502.3
	2	1500	118.0	33.8	17.3	135.9	2.9	506.2
	3	2000	117.3	32.3	17.8	133.0	3.0	510.2
	4	2500	116.7	30.8	18.8	130.4	3.0	513.5
	5	3000	116.1	29.3	19.5	127.8	3.1	517.5
	6	3500	115.5	27.9	20.3	125.3	3.1	521.0
2  11 mph	7	1000	116.0	37.0	18.9	139.6	2.9	508.7
	8	1500	115.2	35.4	19.7	136.9	2.9	512.5
	9	2000	114.4	34.0	20.4	134.3	3.0	516.6
	10	2500	113.7	32.6	21.2	131.7	3.0	521.0
	11	3000	113.1	31.2	21.9	129.2	3.0	525.0
	12	3500	112.4	29.8	22.6	126.8	3.1	529.4
3  13 mph	13	1000	117.4	35.8	21.3	138.4	2.9	505.9
	14	1500	116.4	34.0	22.2	135.4	2.9	510.3
	15	2000	115.4	32.6	23.0	132.5	3.0	516.4
	16	2500	114.6	31.1	23.7	129.6	3.0	519.3
	17	3000	113.7	29.6	24.4	126.9	3.1	524.8
	18	3500	112.9	28.1	25.2	124.3	3.1	528.5

Table 2. Solution for each case. The highlighted rows indicate the most reliable solutions.

The 18 cases shown in Table 2 represent possible solutions when taking into account the uncertainties regarding the initial conditions and the wind speed and direction. However, some solutions are more reliable than others as discussed next, where descriptive geometry and video analysis are brought into the discussion.

3.5 Reliability of the solutions

To select the most reliable solutions, the maximum height (H) reached by the ball in each of the 18 trajectories obtained (Table 2) is compared to the height shown in the video after performing a descriptive geometry analysis using orthogonal and conical projections. A detailed explanation of this analysis is presented in the Appendix. The results indicate that 11 of the 18 calculated trajectories have maximum heights which fall out of the video frames. Thus, these trajectories are discarded since in the video the ball, at its highest point, is always within the frames throughout the filming, i.e. the cameraman never lost sight of the ball until the moment it impacted the deck.

The most reliable solutions whose maximum heights remain inside the video are the following: Solutions #5 and #6 for wind hypothesis 1, solutions #11 and #12 for wind hypothesis 2, and solutions #16, #17 and #18 for wind hypothesis 3. These are highlighted in Table 2. The ESPN's maximum height is also outside the video frame as shown in the Appendix and therefore it is not a reliable solution.

The essence of the analysis is that of the 18 solutions that were found based on different assumptions about spin and wind, only 7 had a maximum height that would have stayed in the view of the camera.

3.6. Seven most reliable solutions for Galarraga home run

Figure 5 shows the trajectory (side and top view) of the ball for each of the seven most reliable solutions indicated in Table 2. The grey area shows the remaining eleven solutions. The curvature of the ball shown on the top view is the result of the combined action of the wind and the sidespin. Considering only the seven most reliable solutions, the total distance (D) of Galarraga's home run is found to be between 517.5 and 529.4 ft (Table 2); all of them exceed 500 feet. The mean value of the total distance of the seven solutions is 523.6 feet.

The total distance of 468 feet found by ESPN is lower than those found in this study, primarily because they used an impact point in row 20 that is 15 feet lower and 9 feet closer to the plate than the distances obtained with LIDAR technology. LIDAR is more reliable because it corresponds to the built stadium and not to a scale model. In addition, ESPN used a longer flight time of 4.97 seconds compared to the value found by this study of 4.67 seconds which was verified using three independent chronometers.

4. Conclusions

A detailed analysis has been made of Andres Galarraga's home run of May 31, 1997. Two distinct claims were previously reported about the distance traveled by Galarraga's home run: the initial estimate of 529 feet by Florida Marlins staff and the more recent claim of 468 feet by Greg Rybarczyk in ESPN Home Run Tracker. In this study, the combined use of four disciplines—physics, geomatics (LIDAR technology), descriptive geometry, and video analysis—proved to be valuable in finding reliable results.

A 3D mathematical model was developed and validated using actual data from Statcast. Eighteen cases were analyzed in order to take into account the uncertainties regarding the initial conditions (exit speed, back and side spins, launch and spray angles) and the wind speed and direction. The seven most reliable solutions were selected by comparing the maximum height of the ball in each analyzed case to the actual height shown in the video frame after applying descriptive geometry

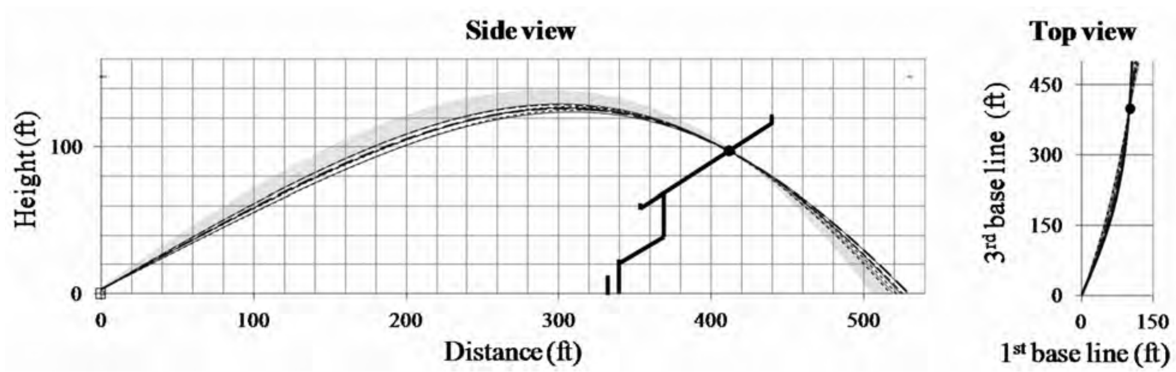


Figure 5. Side and top view of the calculated trajectories for the seven most reliable solutions (Table 2) which are indicated by the dark lines. The remaining eleven solutions are shown in the grey area.

techniques. The accuracy of the solutions was validated by comparing the trajectories predicted by the model with the trajectory shown by the video.

As a result the total distance of Galarraga's home run is estimated to be between 517.5 and 529.4 feet, with a more probable value of 524 feet. This distance range is greater than the value of 468 feet given by the ESPN Home Run Tracker and closer to the value of 529 feet given originally by the stadium's staff. The distance found by ESPN is shorter than those found in this study primarily because their coordinates of the point of impact are 9 feet closer (horizontally) to the plate and 15 feet lower (vertically) than the coordinates obtained in this study, even though in both cases the ball's impact point was on row 20 of the deck. The coordinates in this study were obtained with LIDAR technology which is considered to be more reliable because it corresponds to the actual built stadium and not to a scale model as used by ESPN. Furthermore, the uncertainties regarding the initial conditions and wind were not considered by ESPN. The home run of Galarraga is one of the few hit prior to Statcast proven to have exceeded the 500-foot distance in the history of MLB. ■

HOME RUN DISTANCE

The phrase "home run distance" doesn't mean what many people assume it means. Two separate stats are used during MLB broadcasts. Statcast refers to "hit distance" (DST) as the straight line distance from home plate to whatever the ball first hits, whether seat, fan, or foul pole. But that isn't the number shown when Statcast reports the footage on a home run. This stat, which they refer to as "projected home run distance" (HR-DIS), answers the implied question: how far would the ball have traveled *if the stadium had not been in the way*? From the Statcast glossary on MLB.com: "Projected Home Run Distance is a pivotal tool when comparing individual home runs. Looking at Hit Distance alone is not an optimal practice... [C]omparing the distances of monstrous home runs has long been a hobby of baseball fans. And Projected Home Run Distance gives us a slightly fairer way to do that."

– Cecilia Tan

NOTES

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Appendix

An appendix to this paper, found at https://sabr.org/node/47841, describes in detail the procedure followed to obtain the most reliable solutions for the trajectory and projected distance of Galarraga's home run, accompanied by additional graphs, graphics, and plots. The maximum height (H) reached by the ball in each of the 18 trajectories obtained is compared to the height shown in the video after performing orthogonal and conical projections.

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